Spatial Autoregressive Models for Resource Demand Prediction in Mobile Wireless Networks*

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Mobile wireless networks present unique challenges to the development of tractable models that can incorporate both spatial and temporal correlations in demand induced by subscriber mobility. Space-time autoregressive time series modeling is a promising inductive method that uses a small number of parameters and can be used for online monitoring and load prediction. In this paper, we develop space-time autoregressive models for several wireless network scenarios. We evaluate the ability of the space-time autoregressive models to model the spatial and temporal correlations in the network and show that for the scenarios depicted, the space-time models perform well.

1. Introduction

The recent rapid growth in the number of wireless applications, along with the expectation that wireless networks support the same high quality of service provided by wireline networks, present major challenges to service providers who must support these high quality services in a seamless fashion at a reasonable cost. Software radio introduces the ability to dynamically control the wireless interface and is expected to have a huge impact on performance. Ensuring QoS will require innovative solutions with respect to the development of new architectures and protocols, and will provide unique challenges in developing tractable models that can incorporate both spatial and temporal correlations in demand induced by subscriber mobility.

The cellular architecture has been developed to maximize spectrum utilization. Early analytic models focused on the behavior of a single cell [1–3] based on assumptions that arrival traffic was Poisson and homogeneous, mobility patterns were random, and cell sojourn times were exponentially distributed. Models developed under these assumptions provided a tractable analysis and produced reasonably accurate results for first generation wireless networks.

Since there exists no product form solution for multiple cell topologies and the resulting state space explosion prohibits an exact analysis of the network [4,5], Kelly [4] proposed a fixed-point approximation (FPA) based on the Erlang blocking formula. This method provided a good approximation for blocking probabilities in large networks consisting of multiple cells with low subscriber mobility [6], but has been shown not to be accurate when spatial correlations are strong [7–9].

There has been significant research activity in the utilization of the temporal and spatial characteristics of the physical medium to improve network capacity [10–13]. In this paper we take on a more global approach and concentrate on the modeling and prediction of bandwidth demand as a result of subscriber mobility that is spatially and temporally correlated over a finite time horizon.

To ensure QoS, a number of schemes, most based on priority and admission control, have been proposed. These models typically describe the steady state of the network and enact congestion control or admission policies based

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on the current state of the network. In this paper we propose a method of predicting the future network state from which we can formulate network management policies.

Autoregressive time series analysis is a powerful inductive modeling tool used to forecast resource demand based on real-time measurements over a finite time horizon. Moreover, these models can readily be identified based on empirical measurements using Kalman filtering [14], least squares methods, maximum likelihood estimates (MLE), or through the use of artificial neural networks [15]. However, autoregressive time series analysis models of networks are generally limited to temporal models [16,17].

In [18,19], Pfeifer and Deutsch present a family of multi-variate autoregressive moving average models called space-time autoregressive integrated moving average models (STARIMA). Based on the work of Martin and Oeppen [20], they can capture both temporal and spatial relationships in systems. They have been shown to be a powerful tool in developing parsimonious prediction models in a number of diverse applications from predicting river levels to automobile traffic modeling [18,19,21–23].

With the general deployment of 3G wireless systems, empirical data, both temporal and spatial, are becoming available. In this paper we conduct a number of controlled experiments for several simple network topologies and mobility patterns to test the applicability of STARIMA modeling in wireless environments. We study mobility in a feed forward convergent network, general random mobility in a symmetrical network, and a network with a finite subscriber population. We evaluate the applicability of these models in a controlled environment before applying the models to empirical data.

The rest of this paper is organized as follows: In section 2, we review the STARIMA techniques used in our performance model. In section 3, we present the development of finite-horizon autoregressive models for several simple topologies and mobility patterns. In section 4, we present some numerical results for several call arrival, call holding, and dwell distributions under various mobility patterns. The conclusion is given in section 5.

2. Time Series Models

In this section we review the STARIMA models presented by Pfeifer and Deutsch [18,19,21,22] that are based on common autoregressive models [24].

2.1. STARIMA Process

The space-time ARIMA (STARIMA) class of models presented by Pfeifer and Deutsch [19], and denoted by STARIMA($\boldsymbol{p}_{\lambda_1,\lambda_2,\ldots,\lambda_p}, d, \boldsymbol{q}_{m_1,m_2,\ldots,m_q}$), is defined in vector form as

$$\Delta_d \boldsymbol{z}(t) = \sum_{k=1}^{p} \sum_{l=1}^{\lambda_k} \phi_{kl} \boldsymbol{W}^{(l)} \Delta_d \boldsymbol{z}(t-k) - \sum_{k=1}^{q} \sum_{l=1}^{m_k} \psi_{kl} \boldsymbol{W}^{(l)} \boldsymbol{\epsilon}(t-k) + \boldsymbol{\epsilon}(t). \tag{1}$$

Here, p and q are the autoregressive and moving average orders respectively; λ_k and m_k are the spatial orders of the kth order autoregressive and moving average terms respectively; $\Delta_d \boldsymbol{z}(t)$ is the vector of dth order differences of the observations $\boldsymbol{z}(t)$; ϕ_{kl} and ψ_{kl} are the kth order time lag and lth order spatial lag autoregressive and moving average parameters, respectively; $\boldsymbol{W}^{(l)}$ is the lth order weight matrix; and $\boldsymbol{\epsilon}(t)$ is a vector of random normal errors defined as

$$E[\boldsymbol{\epsilon}(t)\boldsymbol{\epsilon}(t+s)' = \begin{cases} \sigma^2 \boldsymbol{I}_N & s = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The STARIMA process is stationary if every x_u that solves

$$\det \left[x_u^p \boldsymbol{I} - \sum_{k=1}^{p} \sum_{l=0}^{\lambda_k} \phi_{kl} \boldsymbol{W}^{(l)} x_u^{p-k} \right] = 0 \tag{3}$$

lie inside the unit circle. If the process is not stationary the STARIMA process can be transformed to an STARMA process by the application of the difference operator.

The space-time covariance function is

$$\gamma_{lk}(s) = E\left\{ \frac{[\boldsymbol{W}^{(l)}\boldsymbol{z}(t)]'[\boldsymbol{W}^{(l)}\boldsymbol{z}(t+s)]}{N} \right\} \tag{4}$$

and the space-time autocorrelation function is given as

$$\rho_l(s) = \frac{\gamma_{lk}(s)}{[\gamma_{ll}(0)\gamma_{kk}(0)]^{1/2}}. \tag{5}$$

The space-time partial correlations are

$$\gamma_{h0}(s) = \sum_{j=1}^{k} \sum_{l=0}^{\lambda} \phi_{jl}\gamma_{hl}(s-j). \tag{6}$$

In this paper we use the conditional (MLE) function as proposed in [18]. The conditional likelihood function of $\boldsymbol{\Phi}$, $\boldsymbol{\Theta}$ and σ^2 is

$$L(\boldsymbol{\Phi}, \boldsymbol{\Theta}, \sigma^2|\boldsymbol{z}) = (2\Pi)^{-TN/2}(\sigma^2)^{-TN/2} \exp\left(-\frac{S_*(\boldsymbol{\Phi}, \boldsymbol{\Theta})}{2\sigma^2}\right), \tag{7}$$

where $S_*(\boldsymbol{\Phi}, \boldsymbol{\Theta})$ is the conditional sum of squares function

$$S_*(\boldsymbol{\Phi}, \boldsymbol{\Theta}) = \hat{\epsilon}'\hat{\epsilon}. \tag{8}$$

The vector $\hat{\boldsymbol{\epsilon}}$ is calculated using the expression

$$\boldsymbol{\epsilon}(t) = \boldsymbol{z}(t) - \sum_{k=1}^{p}\sum_{l=1}^{\lambda_k} \phi_{kl}\boldsymbol{W}^{(l)}\boldsymbol{z}(t-k) + \sum_{k=1}^{q}\sum_{l=1}^{m_k} \psi_{kl}\boldsymbol{W}^{(l)}\boldsymbol{\epsilon}(t-k), \tag{9}$$

where $\boldsymbol{z}(t)$ and $\boldsymbol{\epsilon}(t)$ are set to 0 for $t < 1$. The MLE's of the sample values $\hat{\sigma}^2$, $\hat{\boldsymbol{\Phi}}$, and $\hat{\boldsymbol{\Theta}}$ are

$$\hat{\sigma}^2 = \frac{S_*\left(\hat{\boldsymbol{\Phi}}\hat{\boldsymbol{\Theta}}\right)}{TN}, \tag{10}$$

where T is the order of the time samples and N is the order of the spatial samples.

The weight matrix $\boldsymbol{W}^{(l)}$ of order l is used to establish the adjacency of sectors and cells. A first-order weight matrix establishes neighboring cells that share a common border, a second-order matrix establishes regions having a common border with neighboring cells but not with the original cell itself, etc. A crucial step in model development is the determination of the weight matrices.

3. Model Development

The evaluation of these models as being appropriate for mobile wireless networks will be based on the method suggested in [19], also known as the Box Jenkins Method [24] which we outline below.

Identification Stage: The first step is to determine whether the space-time series of observations obtained from empirical measurements are stationary. This can be determined by computing the space-time autocorrelation function (ACF) and partial space-time autocorrelation function (PACF) or by using Kalman filtering or other techniques

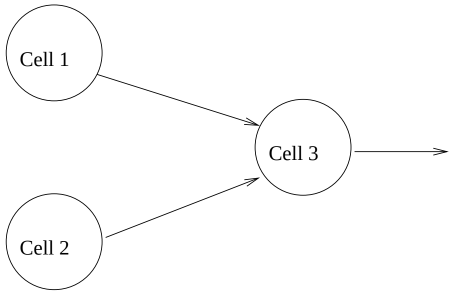


Fig. 1. 3 cell example

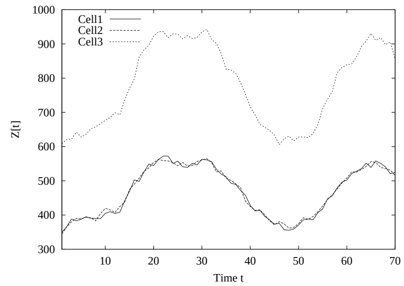


Fig. 2. Simulation data for 3 cells

on the data. If the process is not stationary, then the backshift operator can be used to obtain the second-order differences (differences of the differences). Then the ACF and PACF are recalculated and the backshift operator is applied until they display stationarity. Depending on the values of the ACF and the PACF derived from equations (5)and (6), a candidate model is chosen. This can either be a STAR, STMA, or STARMA or STARIMA model.

Estimation of Parameter Stage: After a candidate STARIMA model is chosen, the parameters ϕ and ψ are fitted using MLE estimates using equations (7)-(10).

Diagnostic Checking Stage: The model will be checked against the actual data and simulation models to see that it accurately represents the dynamics of the observed network behavior. Also, if the model is unduly complex or if the model does not adequately fit the planning horizon, adjustments may need to be made. Additionally, we check the autocorrelations and the partial autocorrelations of the residuals to make sure that they are not significant.

4. Numerical Results and Discussion

In this section we study a few common network topologies and simple mobility patterns to evaluate the ability of the space-time autoregressive models to model the spatial and temporal correlations in the system, and assess how good the prediction is based on the model we built. The mobility patterns we study are basic mobility models commonly found in the literature. Analysis of empirical measurements taken for automobile traffic [23] show that higher order weight matrices are not usually necessary in constructing accurate models, so we restrict our studies to first- and second-order neighbors.

4.1. Experimental Setup

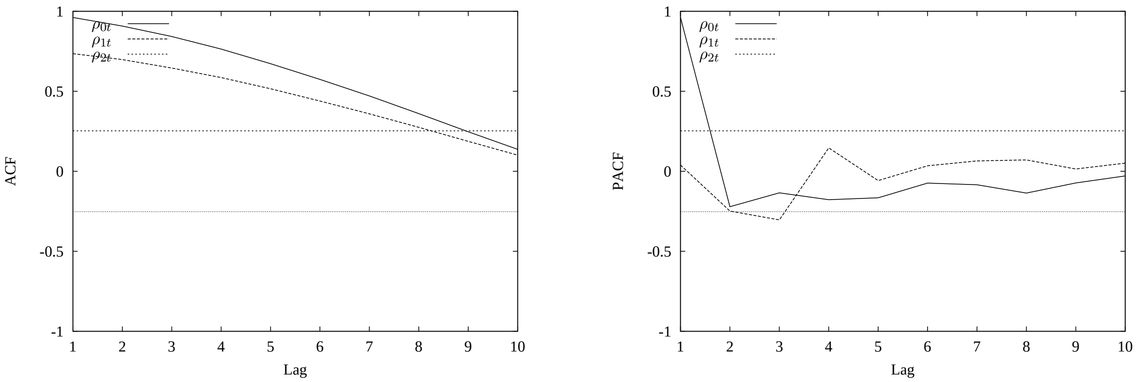
Synthesized traffic traces are generated that represent non-stationary system behavior over a finite time horizon. We model call arrivals as steadily increasing, which can be expressed by a univariate time series: $X_t = 1.8X_{t-1} - 0.8X_{t-2} + \epsilon_t$, where X_0 is the starting number of calls, and ϵ_t is normal (0,1). Subscribers within all cells move at the same constant velocity, with a mean dwell time in a cell which varies in the different scenarios from 15 to 60 seconds. The dwell time for new calls in a cell will be uniformly distributed. Call holding times are Pareto distributed with a mean value of 100s to account for heavy-tailed effects from subscribers sending and receiving data. The number of call requests, both new and handoff, for each cell is sampled every 30 seconds

4.2. Convergent Network

Fig. 1, depicts a small 3 cell network where cell 1 and cell 2 border with cell 3. All 3 cells have the same traffic pattern, but calls in cell 1 and cell 2 can be handoff to cell 3. The starting point for the call arrival process is X_0 = 2, and the dwell time is 15s. The original data trace is shown in Fig. 2. Prediction coefficients are generated using 60 samples. The zeroth- and first-order weight matrices are

$$\boldsymbol{W}^{(0)} = \boldsymbol{I}, \text{ and } \quad \boldsymbol{W}^{(1)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0.5 & 0 \end{bmatrix}.$$

Fig. 3 shows the spatial ACF and PACF for the zeroth and first spatial order. The autocorrelation function decays



(a) ACF (b) PACF

Fig. 3. Spatial ACF and PACF

very slowly, which suggest that it may not be stationary. We apply the difference operator on the data and then compute the spatial ACF and PACF again, the result is shown in Fig. 4. The base lines are 95% confidence bands which are used to judge the correlation cut off values.

From Fig. 4, we can see that the PACF at spatial lag 0 and 1 both cut off after lag 2. We identify this as STAR model with the following autoregressive coefficients.

$$Y[i] = \phi_{01}\boldsymbol{W}^{(0)}\boldsymbol{Y}[i-1] + \phi_{11}\boldsymbol{W}^{(1)}\boldsymbol{Y}[i-1] + \phi_{12}\boldsymbol{W}^{(1)}\boldsymbol{Y}[i-2] + \phi_{13}\boldsymbol{W}^{(1)}\boldsymbol{Y}[i-3] + \epsilon[i] \tag{11}$$

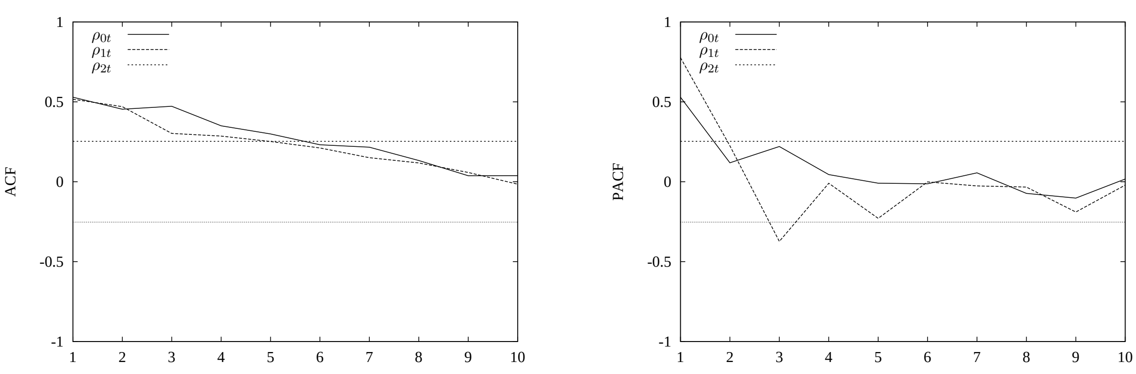
where $\boldsymbol{Y}[i]$ = $\boldsymbol{Z}[i]$-$\boldsymbol{Z}[i-1]$, $\boldsymbol{Z}[i]$ and $\boldsymbol{\epsilon}[i]$ are vectors of observations and errors.

Using equations (7)-(10) to get the MLE estimates, the values for the autoregressive parameters are ϕ_{01} = 0.3627 , ϕ_{11} = 0.6397, ϕ_{12} = 0.3993, and ϕ_{13} = -0.2598.

With the parameters estimated, we check the residual error's spatial autocorrelation and partial autocorrelation as shown in Fig. 5. All values smaller than the 95% confidence interval threshold can be viewed as 0, showing that the model is a good fit.

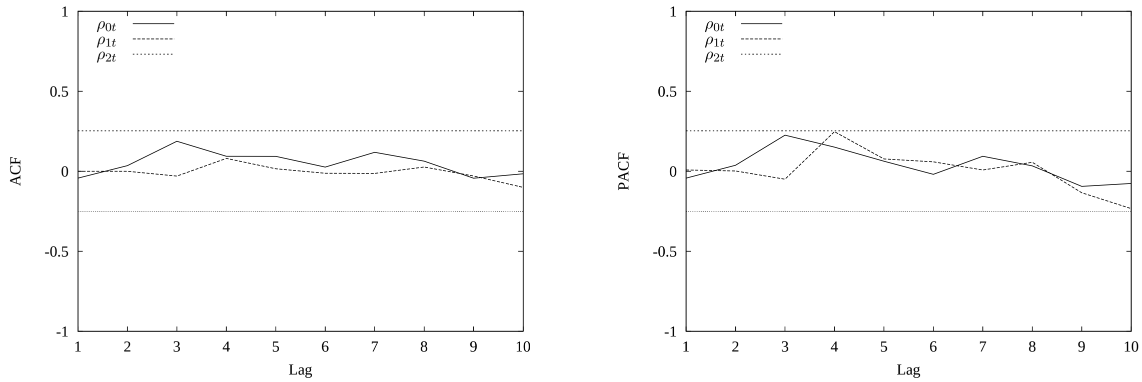
We use the model to generate a 10 step (300s) prediction as shown in Fig. 6. The dotted line is the point prediction data, the solid line is for the original data, and the points are the 95% prediction interval bounds computed as $\hat{Z}_{60}[i] \pm 1.96\sqrt{i\sigma^2}$, (see [25].)

From Fig. 6 we can make a number of conclusions: (1) The prediction reflects the basic trend of the data, (2) The multi-step prediction curve is smoother than the real data, thus predicting general trends, (3) All the real data falls within the prediction intervals, by which we can conclude that the prediction is good, (4) The prediction error increases with an increase in prediction steps.



(a) ACF (b) PACF

Fig. 4. Spatial ACF and PACF of differentiated data



(a) ACF (b) PACF

Fig. 5. Spatial ACF and PACF for the residual error

4.3. Symmetrical Network Case

For the second scenario, we study a 9 cell symmetrical network as depicted in Fig. 7. User mobility is symmetrical with equal probability of visiting adjacent cells.

Call arrivals and call holding times are the same as for the 3 cell scenario The mean dwell time is 60s, and the samples times are averages whereas snapshots were taken in the first scenario. The simulation trace is shown for two neighboring cells numbered 1 and 2 (Fig. 8).

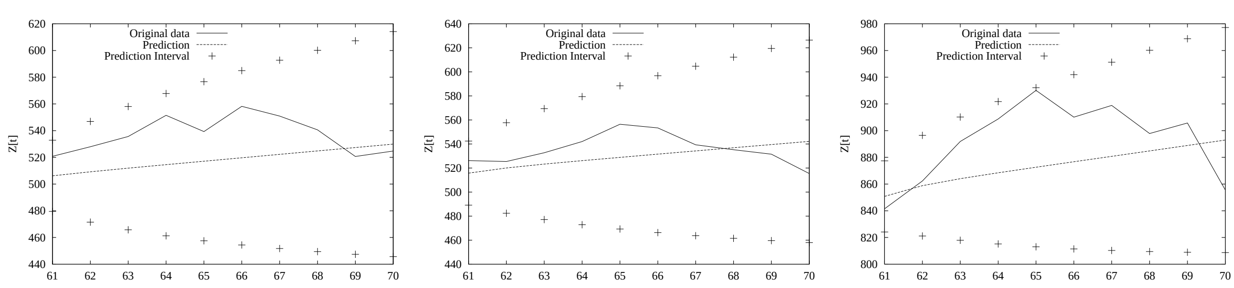
For this scenario, the zeroth- and first-order weight matrices are

$$\boldsymbol{W}^{(0)} = \boldsymbol{I}, \text{ and } \quad \boldsymbol{W}^{(1)} = \begin{bmatrix} 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 1/4 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 0 \\ 1/4 & 1/4 & 0 & 0 & 1/4 & 0 & 1/4 & 0 & 0 \\ 0 & 1/6 & 1/6 & 1/6 & 0 & 1/6 & 1/6 & 1/6 & 0 \\ 0 & 0 & 1/4 & 0 & 1/4 & 0 & 0 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 0 & 1/4 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \end{bmatrix}.$$

The sampled trace is not stationary, so we apply the difference operator on the data and then compute the Spatial Autocorrelation Function and Partial Autocorrelation Function. The result is shown in Fig. 9. We then subtract the difference mean from the sequence generated and the resulting space time autoregressive model is

$$\begin{aligned} Y[i] \quad = \quad & \phi_{01}\boldsymbol{W}^{(0)}(\boldsymbol{Z}[i-1] - \boldsymbol{Z}[i-2]) + \phi_{02}\boldsymbol{W}^{(0)}(\boldsymbol{Z}[i-2] - \boldsymbol{Z}[i-3]) \\ & + \phi_{11}\boldsymbol{W}^{(1)}(\boldsymbol{Z}[i-1] - \boldsymbol{Z}[i-2]) + \phi_{12}\boldsymbol{W}^{(1)}(\boldsymbol{Z}[i-2] - \boldsymbol{Z}[i-3]) + \epsilon[i], \end{aligned} \tag{12}$$

where $\boldsymbol{Z}[i]$ and $\boldsymbol{\epsilon}[i]$ are vectors of observations and errors respectively.



(a) Cell 1 (b) Cell 2 (c) Cell 3

Fig. 6. 10 step prediction

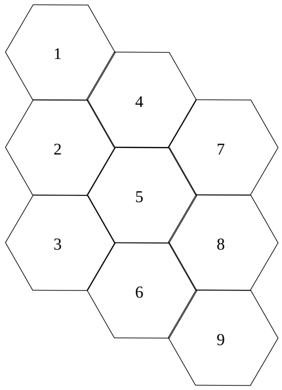


Fig. 7. Symmetrical network

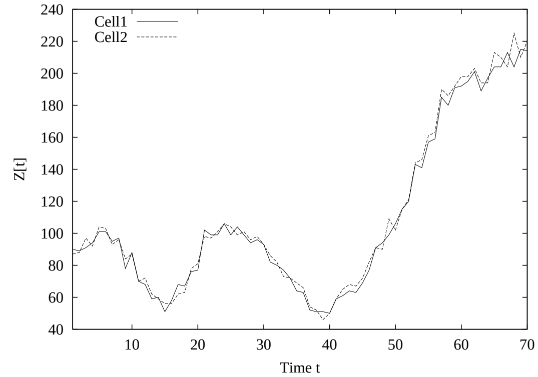


Fig. 8. Symmetrical network data for cell 1 and 2

Using equations (7)-(10) to get the MLE estimates, the values for the autoregressive parameters are ϕ_{01} = -0.5409, ϕ_{02} = -0.0315, ϕ_{11} = 0.6053, and ϕ_{12} = 0.6434.

We check the residual error's spatial autocorrelation and partial autocorrelation, and find that they are sufficiently close to 0 and we generate a 10 step prediction. The results for cell 1 and 2 are shown in Fig. 10.

4.4. Finite Population Scenario

In this scenario, we study a small network with a finite population. We Assume there are 100 subscribers in the network depicted in 11. Unidirectional mobility about a ring [7] is considered. The mean dwell time is exponentially distributed with mean of 100 seconds, samples are taken every 20 seconds.

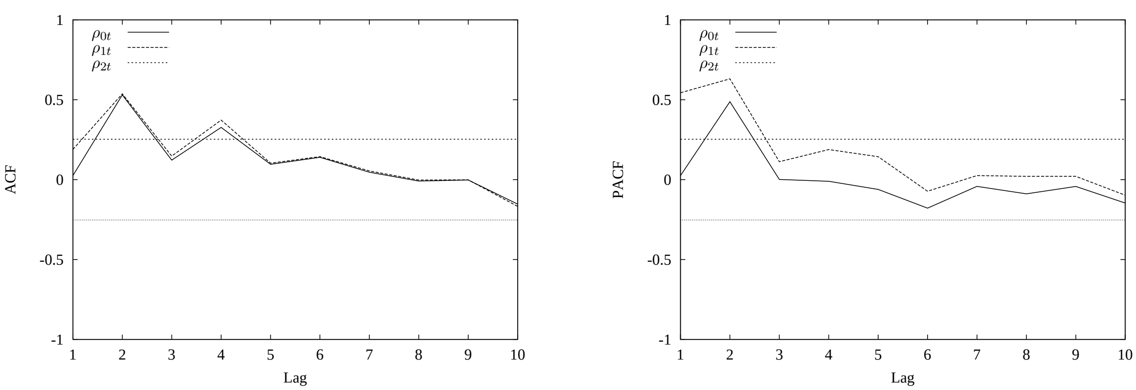
The zeroth-, first- and second-order weight matrices are

$$\boldsymbol{W}^{(0)} = \boldsymbol{I}, \quad \boldsymbol{W}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{W}^{(2)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The simulation data trace for the number of subscribers in cell 2 and cell 3 are shown in Fig. 12. Similar to the computation for spatial ACF and PACF in the previous scenarios, we can see that the data is non-stationary, so we take the difference and then compute the ACF and PACF again. From Fig. 13, we can conclude that the second spatial order is not important. So, the differentiated data can be modeled as a $STAR_1(2)$ model characterized as

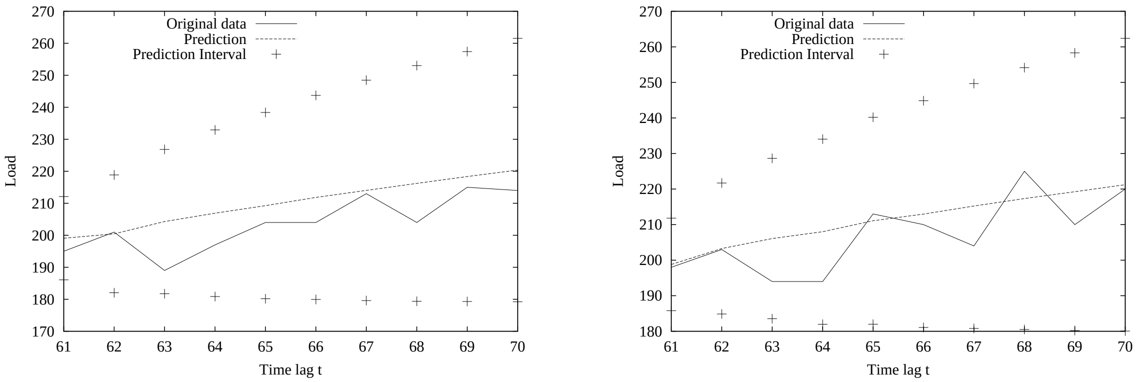
$$\begin{aligned} \boldsymbol{Y}[i] &= \phi_{01}\boldsymbol{W}^{(0)}(\boldsymbol{Z}[i-1] - \boldsymbol{Z}[i-2]) + \phi_{02}\boldsymbol{W}^{(0)}(\boldsymbol{Z}[i-2] - \boldsymbol{Z}[i-3]) \\ &\quad + \phi_{11}\boldsymbol{W}^{(1)}(\boldsymbol{Z}[i-1] - \boldsymbol{Z}[i-2]) + \epsilon[i] \end{aligned} \tag{13}$$

where $\boldsymbol{Z}[i]$ and $\boldsymbol{\epsilon}[i]$ are vectors of observations and errors individually.



(a) ACF (b) PACF

Fig. 9. Spatial ACF and PACF for symmetrical network



(a) cell 1 (b) cell 2

Fig. 10. Prediction for symmetrical network

Using equations (7)-(10) to get the MLE estimates, the values for the autoregressive parameters are ϕ_{01} = 0.2800, ϕ_{02} = -0.2858, and ϕ_{11} = 0.2727.

We check the spatial autocorrelation and partial autocorrelation of the residual errors, and find them within acceptable range. The 10 step prediction for cell 2 and cell 3 based on the derived model is shown in Fig. 14. The prediction data is also acceptable both for point prediction and confidence interval prediction.

5. Conclusions

We have used multi-variate autoregressive models at the network level to investigate the dynamic impact of mobility on these networks for several common mobility patterns with the long range goal being the formulation of a model framework that will aid in the development of a new generation of wireless network models. These models are clearly indicated where spatial and temporal correlations exist and can reduce the number of parameters in generating simulation models. Our current work involves identifying model candidates from empirical measurements taken from 3G systems and further developing weight matrices based on functions of known mobility characteristics in the network. Future work will involve the development and assessment of QoS mechanisms that allow bandwidth to be assigned to cells or regions of the network where the prediction models show it is needed

REFERENCES

1. G. Foschini, B. Gopinath, and Z. Miljanic, “Channel cost of mobility,” *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 1272–80, 1993.

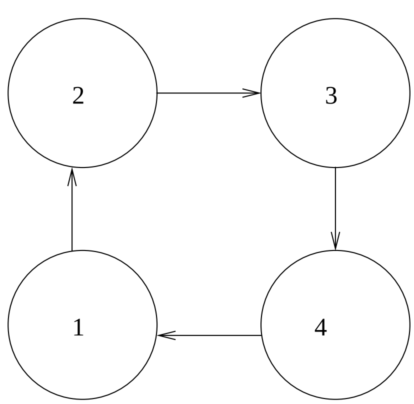


Fig. 11. Finite population example

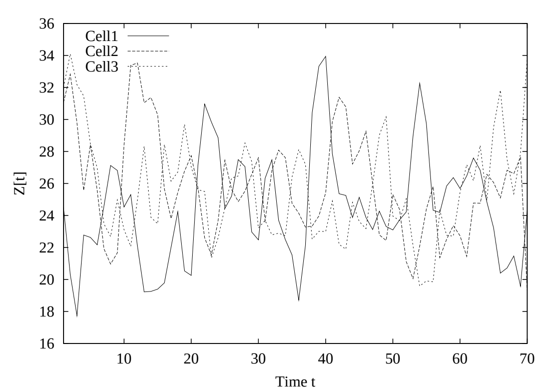
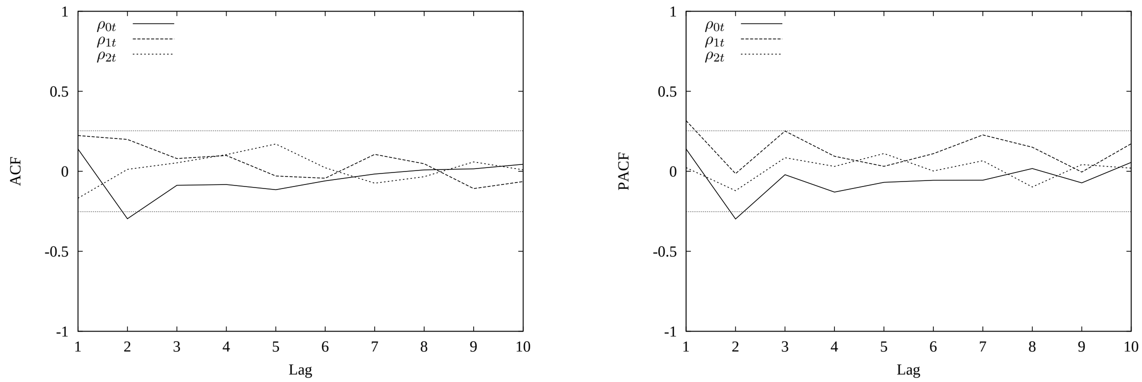


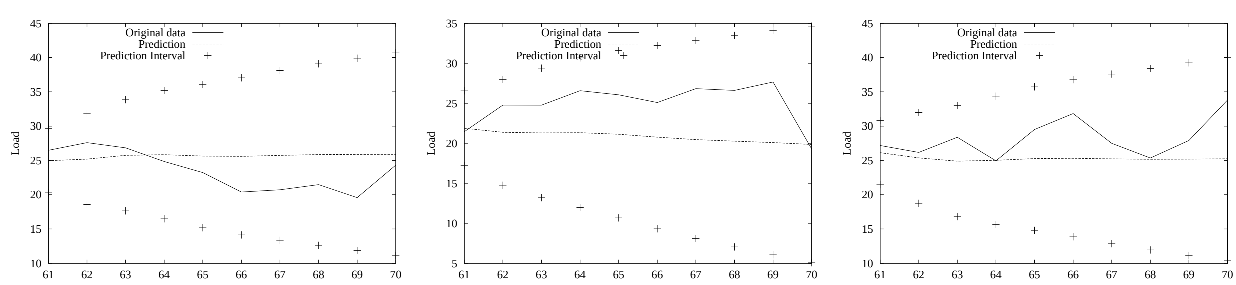
Fig. 12. Finite Population scenario data for cells 1-3



(a) ACF (b) PACF

Fig. 13. Spatial ACF and PACF for finite population scenario

2. D. Hong and S. Rappaport, "Traffic model and performance analysis for cellular mobile radio telephone systems with prioritized and nonprioritized handoff procedures," *IEEE Transactions on Vehicular Technology*, vol. vt-35, no. 3, pp. 77–92, 1986.
3. D. McMillan, "Traffic modeling and analysis for cellular mobile networks," in *13th International Teletraffic Congress*, A. Jensen and V. B. Iversen, Eds. Copenhagen, Denmark: North-Holland, June 1991, pp. 627–632.
4. F. P. Kelly, "Fixed point models of loss networks," *Austral. Math. Soc. Ser. B*, vol. 31, pp. 204–18, 1989.
5. W. Whitt, "Blocking when service is required from several facilities simultaneously," *AT&T Technical Journal*, vol. 64, pp. 1807–56, 1985.
6. K. Mitchell and K. Sohraby, "An analysis of the effects of mobility on bandwidth allocation strategies in multi-class cellular wireless networks," in *Proceedings of the Conference on Computer Communications (IEEE Infocom)*, Anchorage, AK, 2001, pp. 1005–1011.
7. M. Sidi and D. Starobinski, "New call blocking versus handoff blocking in cellular networks," in *infocom*, San Fransisco, California, Mar. 1996.
8. K. Sohraby, "Mobility calculus in cellular wireless networks," in *Conference Record of the International Conference on Communications (ICC)*, Dallas, Texas, June 1996, p. S41.
9. K. Mitchell, K. Sohraby, A. van de Liefvoort, and J. Place, "Approximation models of wireless cellular networks using moment matching," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 11, pp. 2177–2190, 2001.
10. K.Tantinarawal, A.Sayeed, and P.Ramanthen, "Modal space-time channel decomposition for dynamic resource allocation in wireless networks," in *IEEE Sensor Array and Multichannel Signal Processing Workshop*, 2000, pp. 418–22.
11. R.Tingley and K.Pahlavan, "A statistical model of space-time radio propagation in indoor environments," in *IEEE-APS Conference on Antennas and Propagation for Wireless Communications*, 2000, pp. 61–64.



(a) Cell 1 (b) Cell 2 (c) Cell 3

Fig. 14. Prediction for finite population scenario

12. A.Abdi and M.Kaveh, "Space-time correlation modeling of multielement antenna systems in mobile fading channels," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 4, 2001, pp. 2505–08.
13. G.G.Raleigh and T.Boros, "Joint space-time parameter estimation for wireless communication channels," in *IEEE Transactions on Signal Processing*, vol. 46, 1998, pp. 1333–43.
14. R. E. Kalman, "A new approach to linear filtering and prediction problems," *Transactions of the ASME–Journal of Basic Engineering*, vol. 82, no. Series D, pp. 35–45, 1960.
15. R. Feraud, F. Clerot, J. Simon, D. Pallou, C. Labbe, and S. Martin, "Kalman and neural network approaches for the control of a VPbandwidth in an ATM network," in *Proc. of IFIP Networking 2000*, May 2000, pp. 655–666.
16. N. Groschwitz and G. Polyzos, "A time series model of long-term NSFNET backbone traffic," in *Conference Record of the International Conference on Communications (ICC)*, May 1994, pp. 1400–1404.
17. B. Krithikaivasan, K. Deka, and D. Medhi, "Adaptive bandwidth provisioning envelope based on discrete temporal network measurements," in *Proceedings of the Conference on Computer Communications (IEEE Infocom)*, 2004.
18. P. Pfeifer and S. Deutsch, "Identification and interpretation of first order space-time ARMA models," *Technometrics*, vol. 22, no. 3, pp. 397–408, Aug. 1980.
19. ——, "A three-stage iterative procedure for space-time modeling," *Technometrics*, vol. 22, no. 1, pp. 35–47, Feb. 1980.
20. R.L.Martin and J.E.Oeppen, "The identification of regional forecasting models using space-time correlation functions," *Transactions of the Institute of British Geographers*, vol. 66, pp. 95–118, 1975.
21. S. Deutsch and P. E. Pfeifer, "Space-time ARMA modeling with contemporaneously correlated innovations," *Technometrics*, vol. 23, no. 4, pp. 401–409, Nov. 1981.
22. S. Deutsch, M. H. Patel, and A. J. D. Assad, "Comparison of stochastic approaches to the transportation problem: A case study," *IEEE Transactions on Engineering Management*, vol. 41, no. 2, pp. 152–164, May 1994.
23. Y. Kamarianakis and P. Prastacos, "Space-time modeling of traffic flow," in *Proc. European Regional Science Association Conference*, 2002.
24. G. Box and S. Jenkens, *Time Series analysis Forecasting and Control*. San Francisco: Holden-Day, 1976.
25. C. Chatfield, *Principles of Forecasting: A Handbook for Practitioners and Researchers*. Kluwer, 2001, ch. Prediction intervals for time series, pp. 475–494.